

# Evaluation of active structural vibration control strategies in milling process

Monnin, J. (a); Wegener, K. (a)

a) – Institute of Machine Tools and Manufacturing, Zurich, Switzerland

*Keywords:*

*Mechatronics, Milling, Vibration, Control*

## **Abstract**

The ability of different active control strategies to mitigate the apparition of structural oscillations, like forced and self-excited vibrations, of the tool center point during milling process are evaluated and compared using specific measures. These measures quantify the efficiency for vibration suppression by some strategies and the corresponding costs involved. The objective is to be able to identify and dimension the most appropriate control strategies likely to improve the milling process dynamics given an acceptable amount of effort provided by the active system.

First, the definitions of the proposed measures are formulated. A performance measure is defined using the transfer function of the active structure at the tool center point. The effort provided by the actuating system is quantified using the transfer function between the forces coming from the process and those delivered by the actuators. Both measures are then applied to a specific case study in order to evaluate their relevancy and verify their interpretation with regard to the milling process.

The obtained results underline the fact that the chosen measures are able to quantify the improvements of a milling process induced by the implementation of a specific active vibration control strategy and its corresponding effort requirements.

## **1 Introduction**

In milling process it is well-established that forced and self-excited vibrations generated by the cutting interaction constitute the main limiting factors for the machining quality and the productivity. Several methods have been proposed to counteract these phenomena and active vibration controls based on the integration of mechatronic systems into the machine structure is probably one of the most promising. In such solutions, sensors are used to detect the vibrations of the structure and feed the necessary information back to a controller that drive actuators in order to mitigate at the tool center point (TCP) the influence of the disturbances coming from the process. The actual developments in mechatronics dedicated to machine tools are presented by Neugebauer *et al.* [1].

Many different implementations of mechatronic systems can be envisaged as well as, for a given implementation, many strategies can be used by the controller in order to achieve a certain objective. Moreover, different control strategies can produce equivalent levels of performance but with different amount of effort delivered by the actuating system. The idea is to define measures able to quantify the performance and the required effort of a given strategy in order to help its dimensioning and enable a comparison between different controls. A first definition of such measures has been proposed by Monnin *et al.* in [2] considering a simple mechatronic system. Here, these definitions are adapted to a more general model of a machining system.

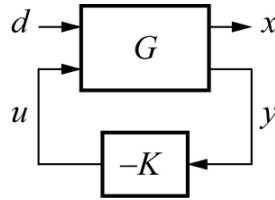
In order to underline the importance of quantitatively comparing different control strategies and of considering effort aspects at the earlier stage of the control design, two different active vibration control strategies are considered. As demonstrated by Ries in [3], active stiffening and active damping controls can positively influence the milling process. More specifically, they can improve the stability against the regenerative chatter apparition leading to a possible increase of material removal rate. Using similar ideas for the quantification, Ries also concludes that active damping strategies are more suitable than active stiffening essentially due to the fact that the amount of energy required by the actuating system is much greater in the second case. This was also confirmed by the results obtained in [2] for a general disturbance rejection objective.

The purpose of this article is thus to define measures capable to quantify the performance and the effort requirement of an active vibration control system dedicated to milling process. The relevancy of these proposed measures is investigated using active stiffening and active damping strategies applied on a

single degree of freedom (SDOF) spring-mass-damper system used to represent the machine structure.

First, the measures are defined. The evaluation of the active structure using these measures is investigated analytically. The milling process is then taken into account and the correspondence between the defined measures and the machining characteristics is verified using numerical simulations.

## 2 Measures definition



**Figure 1:** Block diagram model of the active system

The block diagram shown in figure 1 represents the global system constituted by the machine structure, corresponding to the transfer function  $G$ , and the mechatronic system, represented by the negative feedback  $-K$ . The  $x$  outputs are the deviations of the TCP from its nominal states and  $d$  the disturbing loads generated by the cutting process. The  $y$  outputs represent the signals measured by the sensors of the mechatronic system and delivered to the controller in order to generate the control actions  $u$ .

The dynamics of the machine structure between its two inputs and two outputs is given by the following transfer functions matrix

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} G_{xd} & G_{xu} \\ G_{yd} & G_{yu} \end{bmatrix} \begin{bmatrix} d \\ u \end{bmatrix}. \quad (1)$$

The time variation of the cutting forces  $d$  does not only depend on the nominal process parameters, like the rotational speed, number of teeth, *etc.*, but is also function of the relative deviations of the TCP  $x$ . This closed-loop interaction between the machine and the process is susceptible to give rise to chatter phenomenon. In milling process, also the resulting nominal chip thickness variation

can excite the machine structure and generate forced vibrations between the tool and the workpiece. The ability of the machine structure to resist to these two dynamical phenomena is given by its transfer function between  $d$  and  $x$ .

Thus, the control objective of the active system can be seen as an improvement of the structural characteristics of the machine such that the deviations of the TCP become less influenced by the disturbances  $d$  coming from the process. So the overall performance of the control strategy can be quantified by the maximum of the closed-loop transfer function between the disturbances  $d$  and the outputs  $x$ . For a system with scalar inputs and outputs, one defines a performance measure  $P$  as

$$P = \frac{\|G_{OL}(\omega)\|_{\infty}}{\|G_{CL}(\omega)\|_{\infty}} - 1 \quad (2)$$

where  $\|\cdot\|_{\infty}$  represents the maximum norm and

$$G_k(\omega) = G_k(s) \Big|_{s=i\omega} \quad (3)$$

for  $k = OL, CL$ ;  $s$  corresponding to the Laplace variable and  $i = \sqrt{-1}$ . Also,

$$G_{OL}(s) = \left( \frac{x}{d}(s) \right) \Big|_{open-loop} \quad (4)$$

$$= G_{xd}(s) \quad (5)$$

and

$$G_{CL}(s) = \left( \frac{x}{d}(s) \right) \Big|_{closed-loop} \quad (6)$$

$$= G_{xd}(s) - G_{xu}(s)K(s)S(s)G_{yd}(s) \quad (7)$$

where  $S$  is the output sensitivity equal to

$$S(s) = \left( 1 + G_{yu}(s)K(s) \right)^{-1}. \quad (8)$$

The measure  $P$  represents the relative decrease of the maximum amplitude of the dynamical compliance due to the feedback control. In the case of stable milling process, it can be interpreted as the vibrations amplitude reduction for a machining force exciting the structure at its most critical resonance frequency. From the regenerative chatter point of view and for lightly damped structures, the measure  $P$  approximates the relative increase of the minimal unconditionally stable axial depth of cut visible on the stability lobes diagram (SLD).

A second measure representing the effort requirement of the actuating system is defined as the maximum value of the frequency response function between the controller output  $u$  and the perturbation input  $d$ . This effort measure  $E$  is formally defined as

$$E = \left\| K(\omega)S(\omega)G_{yd}(\omega) \right\|_{\infty} \quad (9)$$

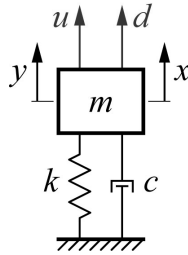
due to the fact that

$$u(s) = -K(s)S(s)G_{yd}(s)d(s). \quad (10)$$

Both proposed measures are quite conservative. They are defined such that they are dimensionless and equal to zero if no closed-loop control is applied. They only refer to open-loop frequency response functions and therefore can be used with experimental data obtained from the open-loop system over a specific bandwidth. Moreover, as both measures depend on the controller parameters, they can be utilized in the optimization controller design phase as well. The extension to multiple-input multiple-output (MIMO) systems can easily be imagined. The main practical inconvenience is that generally the transfer functions between the process disturbance and the TCP are not directly known.

### **3 Case study**

In order to illustrate the approach proposed here, a case study is considered where a SDOF spring-mass-damper system is used to represent the machine structure. First, the results are derived analytically without considering the milling process. Second, a numerical example is used to simulate the machining process coupled with the active structure and make the link with the proposed measures.



**Figure 2:** SDOF spring-mass-damper system

The machine structure is modeled like a simple spring-mass-damper system as shown in figure 2. Its structural parameters are its moving mass  $m$ , the static stiffness  $k$  and its viscous damping coefficient  $c$ . The modal characteristics are the natural frequency  $\omega_n$  and the damping ratio  $\zeta$  defined as

$$\omega_n = \sqrt{\frac{k}{m}}, \quad \zeta = \frac{c}{2m\omega_n}. \quad (11)$$

The structural damping is considered to be sufficiently light so that  $\zeta \ll 1$ . The concentrated mass is excited by the process load  $d$ . The displacements of the mass from its equilibrium point  $x$  represent the deviations of the TCP from its nominal path. For a lightly damped structure, the resonance peak of the dynamic compliance function between the cutting loads and the TCP deviations is located close to the natural frequency and has a magnitude approximately equal to  $1/(c\omega_n)$ .

The control system measures the movements  $y$  of the mass and generates a control force  $u$  acting on the mass in parallel with  $d$ . The dynamics of the sensors, actuators and controller are considered to be perfect, *i.e.* without any time delay.

Different control strategies can be envisaged to reduce the amplitude of the resonance peak of the dynamic compliance function. As previously mentioned, active damping and active stiffening are considered here. For such collocated system, one of the most popular active damping strategies is the direct velocity feedback (DVF) as detailed in [4]. Its popularity stems from its simplicity and efficiency. The feedback loop plays the role of an additional viscous damper. In practice, direct velocity feedback can be achieved in different ways depending on the signal supplied to the controller. Generally, the velocity of the movement is not directly measurable and must be derived from the displacement or from the acceleration. Using displacement signal, a lead compensator must be implemented

into the controller which actually plays the role of a differentiator. However, this might cause problems if large measurement noise is present. With an acceleration signal, the controller must act as an integrator. The implementation of a second order filter into the controller makes the active system acting as an additional tuned-mass-damper system which is nothing but an active damping system targeted on a specific mode of the structure. It presents also the advantage of inducing a greater roll-off than a simple integrator.

In order to show the importance of the choice of the strategy on the performance but also on the effort, an active stiffening control is compared with the active damping. The most straightforward active stiffening control consists in a direct position feedback (DPF). DPF acts like an additional spring element on the moving mass.

For simplicity matter, the case where the velocity of the mass is directly sensed is considered for the DVF. For the DPF, the position is transmitted to the controller. Thus, in both cases, the control filter consists in a scalar static gain, so that

$$K(s) = K_c \quad (12)$$

where  $K_c \in \mathfrak{R}_+^*$ .

In this example, the transfer functions from the controller point of view  $G_{yu}$  and  $G_{yd}$  are identical. Also, from the process point of view, the transfer functions  $G_{xu}$  and  $G_{xd}$  are identical and are always equal to the dynamical compliance function

$$G_{xd}(s) = G_{xu}(s) = G_x(s) = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \quad (13)$$

## 3.1 Analytical example

### 3.1.1 Direct velocity feedback

As already noticed, with this first strategy the transfer function of the machine structure from the controller side is given by

$$G_{yu}(s) = G_{yd}(s) = \frac{s}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \quad (14)$$

Therefore, the transfer function of the closed-loop system between the process disturbance and the TCP deviations becomes

$$G_{CL}(s) = G_x(s)S(s). \quad (15)$$

For lightly damped structures, the maximum amplitude of its frequency response can be approximated by its value at the natural frequency which gives

$$\|G_{CL}(\omega)\|_{\infty} \cong \frac{1}{(c + K_c)\omega_n}. \quad (16)$$

In order to keep the assumption of light damping valid, the values of  $K_c$  must remain in the order of  $c$ .

The resulting performance measure can be approximated by

$$P \cong \kappa_v \quad (17)$$

where  $\kappa_v = K_c / c$ .

In the same manner, the effort measure becomes

$$E_v = \frac{\kappa_v}{\kappa_v + 1}. \quad (18)$$

It is worthwhile to notice that this measure is upper bounded to 1.

Expressed as a function of the performance measure,  $E_v$  can be rewritten as

$$E_v(P) \cong \frac{P}{P + 1} \quad (19)$$

where  $P$ , as a consequence of the assumptions on the damping, must remain in the order of 1.

### **3.1.2 Direct position feedback**

Using a direct position feedback the transfer function of the plant becomes equal to the dynamical compliance function,

$$G_{yu}(s) = \frac{1}{m(s^2 + 2\zeta\omega_n s + \omega_n^2)}. \quad (20)$$

Under the assumption of light damping, the performance measure can be approximated by the expression

$$P \cong \sqrt{\kappa_p + 1} - 1 \quad (21)$$

where  $\kappa_p = K_c / k$ .

The corresponding effort measure is approximated by

$$E_p \cong \frac{1}{2\zeta} \frac{\kappa_p}{\sqrt{\kappa_p + 1}}. \quad (22)$$

By expressing  $E_p$  in function of  $P$ , one finds that

$$E_p(P) \cong \frac{1}{2\zeta} P \left( 1 + \frac{1}{P+1} \right). \quad (23)$$

For both strategies, as the plant is minimum phase, the performance measure is *a priori* not upper bounded. In other words, each strategy is capable to achieve the same level of performance. However, the investigations on the effort show that this level of performance can not be obtained with the same effort. The ratio of both effort measures expressed in term of the performance measure is equal to

$$\frac{E_p(P)}{E_v} \cong \frac{P+2}{2\zeta} \quad (24)$$

where  $P$  is considered up to values in the order of 1. This implies that

$$\frac{E_p(P)}{E_v} > 1, \quad (25)$$

if the assumption of light damping holds.

Although both active control strategies could theoretically achieve the same performance, the effort required by the DPF is always greater for every considered performance level. The difference of the effort depends on the damping ratio of the machine in such a way that the greater the modal damping is, as smaller the difference of effort between both strategies becomes.

This first result confirms the observations coming from preceding studies [2,3] which state that active stiffening strategies requires much more energy than active damping in order to achieve an equivalent vibration mitigation. Thus, the proposed measures seem to correctly quantify the performance and effort characteristics of the active control system.

These observations also emphasize the importance of considering the effort aspect in the control design phase as it can greatly influence the dimensioning of the actuating system.

### 3.2 Numerical example

In this section, the interaction between the active structure and the milling process is simulated using a numerical example. First, the relation between the performance measure  $P$  and the stability lobes diagram is verified. Secondly, the correspondence of the measures with the results of time domain simulations is investigated.

This example is based on a case study proposed by Bayly *et al.* [5] employed to experimentally verify the stability lobes prediction using the time finite element analysis method for highly intermittent milling processes. The same example has been used also by Merdol and Altintas [6] in order to verify the SLD computed by the multi frequency method.

In this case, the machine structure is considered flexible in the feed direction with the modal parameters listed in the table 1.

**Table 1:** Modal parameters of the machine structure

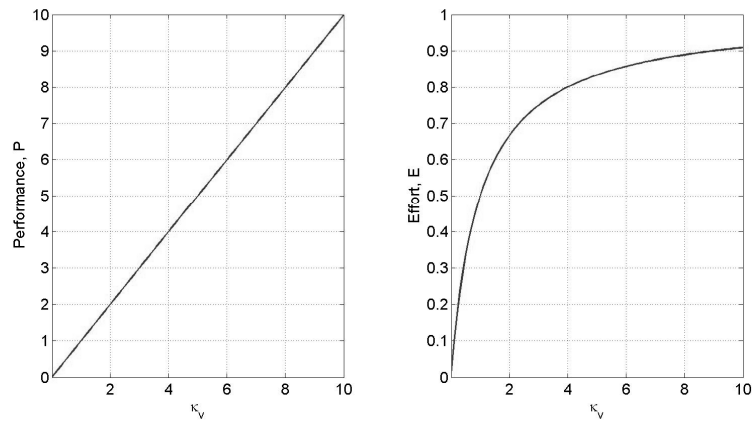
<b>Natural frequency (Hz)</b>	<b>Damping ratio (%)</b>	<b>Modal stiffness (N/<math>\mu</math>m)</b>
146.5	0.32	2.18

A workpiece in aluminum (7075-T6) is up-milled using a 19.05 mm diameter carbide end mill with a single flute. A constant feed of 0.102 mm/rev. is considered. A

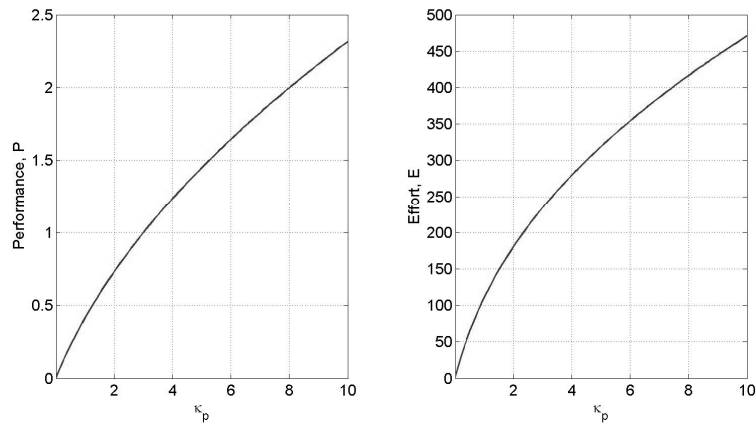
proportional cutting force law model is used with tangential and radial specific cutting pressures equal to  $K_t = 550$  MPa, resp.  $K_r = 200$  MPa.

### 3.2.1 Measures computation

Active damping and stiffening controls are implemented to mitigate vibration problems. Using the parameters of the table 1, the performance and effort measures can be computed over a control gain range. The obtained results are represented in the figure 3 for the DVF and in the figure 4 for the DPF. The correspondence with the relations (17), (18), (21) and (22) is easily verified. The very light modal damping of the structure exacerbates the difference of effort requirement between both strategies, which is obvious here given an equivalent performance.



**Figure 3:** Performance and effort measures obtained with DVF



**Figure 4:** Performance and effort measures obtained with DPF

### 3.2.2 Stability lobes diagrams

For lightly damped structures, the performance measure can be seen as the relative increase of the minimal unstable axial depth of cut visible in the stability lobes diagrams.

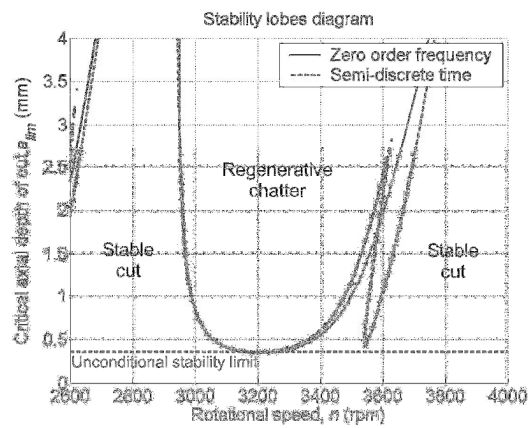
A low radial immersion of  $RDOC^1 = 0.237$  is considered for the computation of the SLD, as presented in the articles [5,6]. Two different methods are used here to compute the stability lobes diagram by taking into account the influence of the active control system on the structural properties of the machine. The first method has been proposed by Altintas and Budak [7] which employs stability criteria based on the frequency domain and approximates the time varying directional matrix by its zero order component (ZOA). This method allows a fast SLD computation but can be inaccurate when the radial immersion becomes small. For highly intermittent cutting, the semi-discrete time domain (SDT) method proposed by Insperger and Stepan [8] provides more accurate predictions but requires also more computational efforts. This method is based on the Floquet theory and is able to detect Hopf but also flip bifurcations.

When no active control is applied on the structure, the stability lobes diagram computed with both ZOA and SDT methods is shown in figure 5.

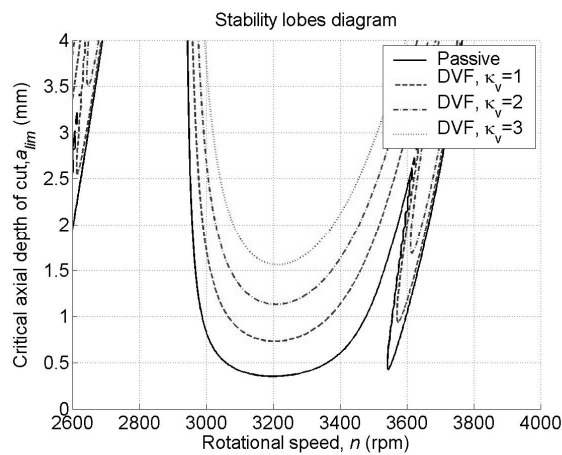
---

<sup>1</sup> RDOC: Radial depth of cut.

Figure 6 represents the evolution of the stability lobes computed with the SDT method and considering a DVF with different values of control gain. The transfer function of the resulting active structure is used for the computation. One observes that the increase of the unconditional stability limit is proportional to the control gain. This underlines the fact mentioned previously that the performance measure can be seen as the relative increase of the minimum critical axial depth of cut which is also confirmed by the observation of the results obtained with the DPF.



**Figure 5:** Stability lobes diagram without any active control



**Figure 6:** Influence of DVF on SLD

### **3.2.3 Time domain simulations**

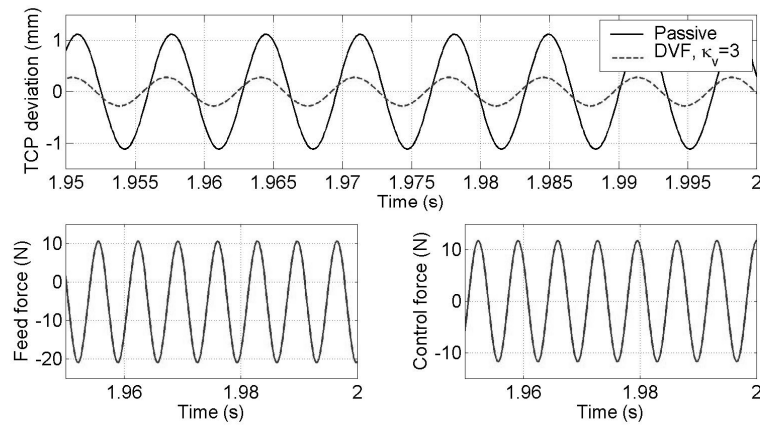
A proprietary code developed in Matlab® is used to run time domain simulations of the milling process coupled with machine structure and the active control system. The experimental results presented in [5] are used for its validation.

In the previous section, the relation between the performance measure and the chatter phenomenon is represented. The performance measure corresponds also to the diminution of the vibration amplitudes at the TCP when the machining process is stable and excites the most critical structural mode around its natural frequency. In the same conditions, the effort measure represents the magnitude ratio between the force generated by the active system and the cutting force.

In order to verify this remark using time domain simulation, machining conditions must be chosen such that the process generates a harmonic force at the resonance frequency. To do so, a slot machining, *i.e.* RDOC = 1, and a cutter with two flutes is considered. The rotational speed of the cutter is chosen equal to 4400 rpm in order to obtain a tooth passing frequency corresponding to the resonance frequency of the structure. In this condition, the structure is in resonance and large vibration amplitudes are observed at the TCP. However, this spindle speed corresponds to a stability peak of the system in the SLD and even if large amplitudes are observed the process remains stable.

The upper plot of figure 7 compares the TCP deviations along the feed direction delivered by the time domain simulation without active control with those using a DVF with a control gain  $\kappa_v = 3$ . For such control gain, the performance measure is equal to 3 and the effort measure is equal to 0.75 according to results indicated in figure 3. One notes that the relative difference of amplitudes of the TCP deviations is equal to the performance measure  $P$ . The lower plots in figure 7 represent the cutting force in the feed direction (left plot) and the control force (right plot) when DVF is applied. The amplitude ratio between both forces corresponds to the effort measure.

These observations confirm the interpretation of the proposed measures.



**Figure 7:** Results of time domain simulations in steady state conditions

## 4 Conclusion

In the present article, two measures are defined in order to quantitatively evaluate, dimension and compare different active control strategies dedicated to the mitigations of vibration problems in milling process. A measure quantifies the positive influence of the control over the structural behavior of the machine. The other one gives a measurement of the effort required by the actuating system to apply the control strategy.

To illustrate the approach and to verify the relevancy of the proposed measures, a case study involving a single degree of freedom spring-mass-damper system is used to represent the machine structure. Active damping and active stiffening strategies are implemented to improve the structural dynamics against the process disturbances.

An analytical study demonstrates the ability of the measures to confirms the fact that active stiffening methods are much more effort demanding than active damping strategies. An interpretation of the measures in term of milling process is shown using the stability lobes diagram and time domain simulations.

This study shows that the proposed measures are susceptible to conservatively but representatively quantify the performance and the effort of an active control system dedicated to milling process. In the presented example, the controller is a single-

input single-output system but the measures can easily be extended to MIMO systems. An important advantage of these measures is that they can be used at the earlier stage of the control design phase and can be used with open-loop experimental frequency response functions. They can also be integrated in optimization scheme for the control dimensioning procedure.

## **Literature**

- [1] Neugebauer, R.; Denkena, B.; Wegener K.: *Mechatronic Systems for Machine Tool*. *Annals of the CIRP* 56 (2007) 2, p. 657-686
- [2] Monnin, J.; Wild, H.; Wegener, K.: *Integration of Effort Aspect into Control Strategy Dimensioning and Comparison*. *Proceedings of the 12<sup>th</sup> Mechatronics Forum Biennial International Conference, Swiss Federal Institute of Technology, ETH Zurich, Switzerland, June 28-30, 2010*
- [3] Ries, M.: *Aktive Motorspindel zur Unterdrückung von Ratterschwingungen im Fräsprozess*. *Dissertation, TU Darmstadt, VDI-Verl. (2009)*
- [4] Preumont, A.: *Vibration Control of Active Structures, An Introduction*. 2<sup>nd</sup> edition, *Kluwer Academic Publishers, Solid Mechanics and its Applications* 96 (2002)
- [5] Bayly, P. V.; Mann B. P.; Schmitz, T. L.; Peters, D. A.; Stepan, G.; Insperger, T.: *Effects of Radial Immersion and Cutting Direction on Chatter Instability in End-Milling*. *Proceedings of IMECE (2002), MED-39116*
- [6] Merdol, S. D.; Altintas, Y.: *Multi Frequency Solution of Chatter Stability for Low Immersion Milling*. *Journal of Manufacturing Science and Engineering* 126 (2004), ASME, p. 459-466
- [7] Altintas, Y.; Budak, E.: *Analytical Prediction of Stability Lobes in Milling*. *Annals of the CIRP* 44 (1995) 1, p. 357-362
- [8] Insperger, T.; Stepan, G.: *Updated Semi-discretization Method for Periodic Delay-Differential Equations with Discrete Delay*. *International Journal for Numerical Methods in Engineering* 61 (2004), p. 117-141